Calculating the Frequency of DCS Signals

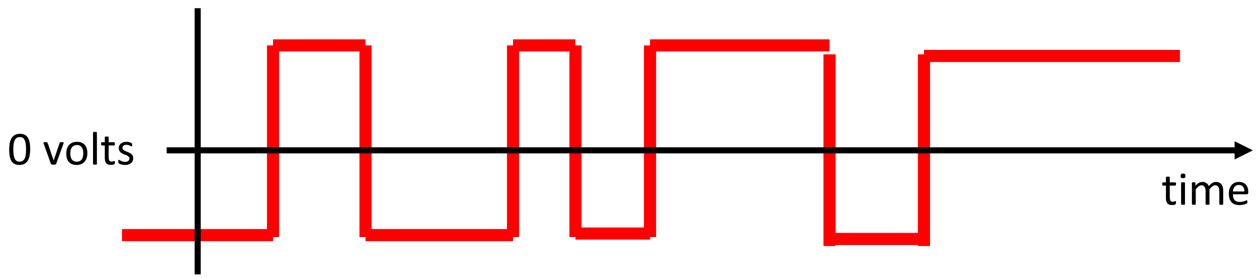
Adrian Tang

Calculating the Frequency of DCS Signals

This document explains how the frequency content of the DCS signal is computed mathematically….

We begin by looking at our DCS signal in time domain which is a polar NRZ signal:

Our DCS signal on the track Y(t)

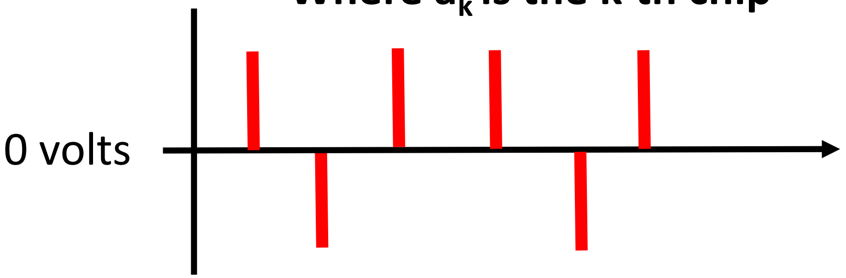


Calculating the Frequency of DCS Signals

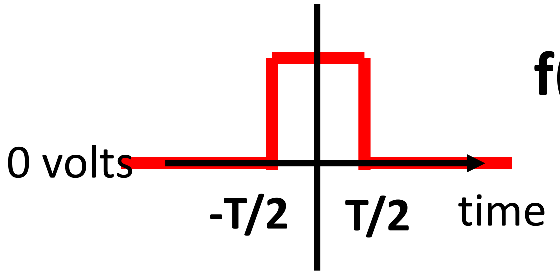
We can actually decompose the DCS signal into two mathematical signals and model the DCS signal on the track as a LTI system with an impulse response that is our DCS pulse, and and an input impulse train with only the polarities

x(t) Impulse train carrying the polarity of our data $x(t) = a_k\delta(t-nT)$

Where a_k is the k'th chip

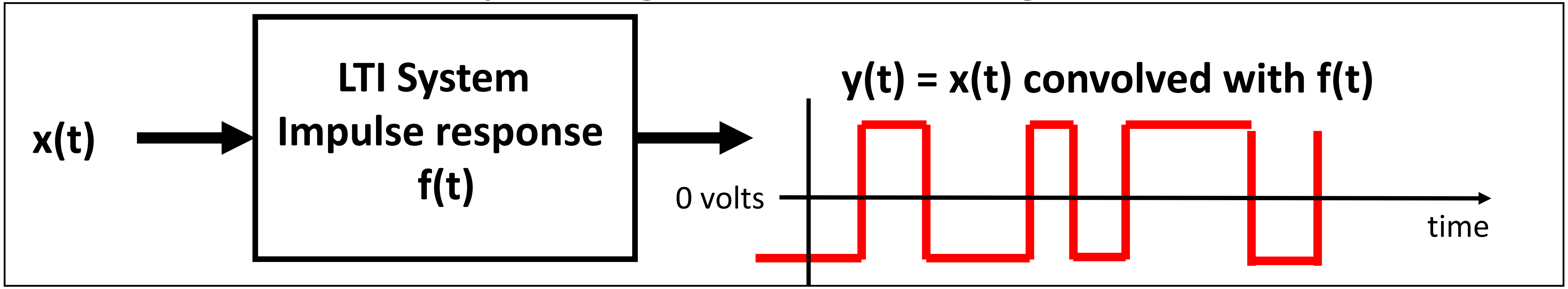


f(t) our DCS pulse without the polarity considered



$f(t)$ **=rect (t/T)**

Decomposed Signal Model of DCS signal



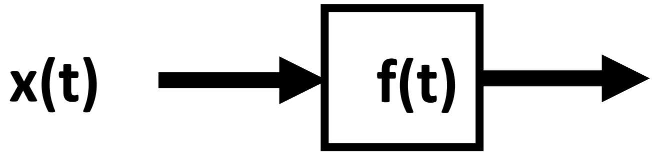
Calculating the Frequency of DCS Signals

With this model we can exploit the fact that convolution in the time domain is multiplication in the frequency domain:

$$\int_{-\infty}^{\infty} [x(t) * f(t)] e^{i\omega t}\, dt = X(\omega) F(\omega)$$

$$Where \int_{-\infty}^{\infty} x(t) e^{i\omega t} dt = X(\omega) \quad and \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt = F(\omega)$$

So our drawing of DCS in frequency domain becomes



$S_y(\omega) = |F(\omega)|^2 S_x(\omega)$

Sy is the thing we are trying to find Which is the power spectral density of the DCS signal

Sx is the power spectral density of x(t)
and Sy is the power spectral density of y(t)

Calculating the Frequency of DCS Signals

Right so we just introduced power spectral density (PSD) which is the frequency content of a given signal. Unlike a fourier transform the PSD is for statistical signals (signals with random sequences like our DCS packet).

→ PSD is formally defined as the Fourier Transform of the autocorrelation of a signal

Auto-correlation is a measure of how similar a signal is to a time shifted version of itself, and is formally defined as:

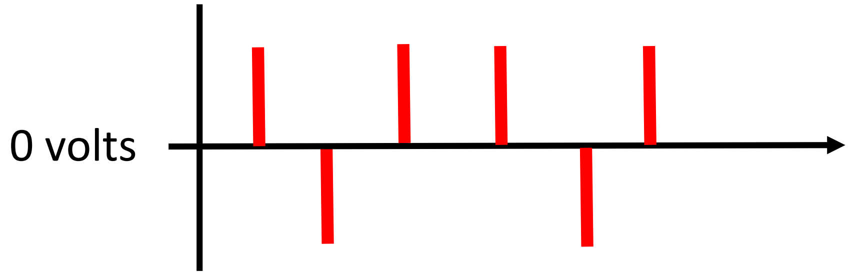
$$Rx(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t-\tau)dt$$

The key important thing is that Sx and Rx are Fourier transform pairs that we need to find the spectrum of our DCS signal

Calculating the Frequency of DCS Signals

So back to x(t)….. We need to figure out **Sx** and so we need his autocorrelation function

x(t) = $a_k\delta(t-nT_b)$



Definition again:

$$Rx(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)x(t-\tau)dt$$

But hey, it's periodically spaced and has no values in-between so we can **turn the integration into a simple sum:**

$$Rx(\tau) = \frac{1}{T}\sum_{N=-\infty}^{\infty}\left[\lim_{N\to\infty}\sum_{K} a_k a_{k+m}\right]\delta(\tau - nT)$$

Calculating the Frequency of DCS Signals

So our autocorrelation of the input is:

$$Rx(\tau) = \frac{1}{T}\sum_{N=-\infty}^{\infty}\left\lfloor \lim_{N\to\infty}\sum_{K} a_k a_{k+n} \right\rfloor \delta(\tau - nT)$$

For real physical signals (like DCS) Rn = R_{-n} so we can collect the sums (except the N=0 term is still extra outside)

$$Rx(\tau) = Ro + \sum_{N=1}^{\infty}\left\lfloor \lim_{N\to\infty}\sum_{K} a_k a_{k+n} \right\rfloor$$

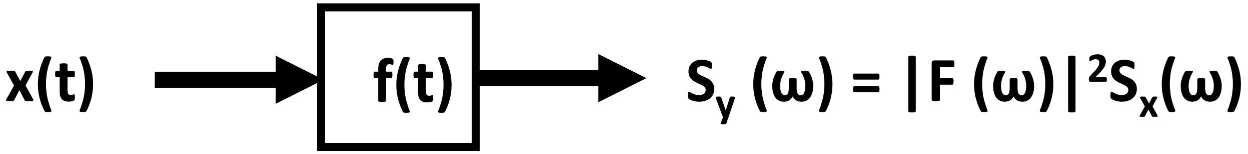
So great, now we can directly exploit those Fourier transform pairs to get Sx

$$Sx(\omega) = \frac{1}{T}\left[Ro + 2\sum_{N=1}^{\infty}\left\lfloor \lim_{N\to\infty}\sum_{K} a_k a_{k+n} \right\rfloor e^{in\omega t}\right]$$

Calculating the Frequency of DCS Signals

$$Sx(\omega) = \frac{1}{T}\left[Ro + 2\sum_{N=1}^{\infty}\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right| e^{in\omega t}\right]$$

So going back to our DCS model drawing:



We can sub in Sx now that we know it and solve for our DCS signal (Sy)…

$$S_y(\omega) = \frac{|F(\omega)|^2}{T}\left[Ro + 2\sum_{N=1}^{\infty}\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right| e^{in\omega t}\right]$$

So now we jut need to apply some basics to simplify this and get a plot going….

Calculating the Frequency of DCS Signals

$$S_y(\omega) = \frac{|F(\omega)|^2}{T}\left[Ro + 2\sum_{N=1}^{\infty}\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right| e^{in\omega t}\right]$$

Right so lets tackle Ro first:

Ro is the case where n=0 so that inside sum simplifies to a_k^2
→So if a_k^2 is -1 or +1 it doesn't matter, since the square is always +1
→ Also there's exactly N values in the sum since we have the same number of pulses as autocorrelation steps…

$$Ro = \lim_{N\to\infty} \frac{1}{N}\sum_{K} a_k a_k = \frac{1}{N}(N)$$

$$Ro = 1$$

Okay cool… so Ro is just 1

Calculating the Frequency of DCS Signals

$$S_y(\omega) = \frac{|F(\omega)|^2}{T}\left[Ro + 2\sum_{N=1}^{\infty}\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right| e^{in\omega t}\right]$$

Right so lets tackle the other term next:

$$\sum_{N=1}^{\infty}\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right|$$

So a_k and a_{k+n} are assumed totally uncorrelated and through a long of DCS command you would expect roughly the same number of 0 and 1 chips which are +1 and -1 (since DCS is centered around 0 volts). That means roughly half the products will be 1 and half the products will be -1 for every pair of k and k-n in N=1 to infinity sets:

$$\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right| = \lim_{N\to\infty}\frac{1}{N}\left|\frac{N}{2}(1) + \frac{N}{2}(-1)\right|$$

$$\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right| = 0$$

Calculating the Frequency of DCS Signals

So back to our equation for the DCS power spectral density

$$S_y(\omega) = \frac{|F(\omega)|^2}{T}\left[Ro + 2\sum_{N=1}^{\infty}\left|\lim_{N\to\infty}\sum_{K} a_k a_{k+n}\right| e^{in\omega t}\right]$$

We just showed it simplifies down to

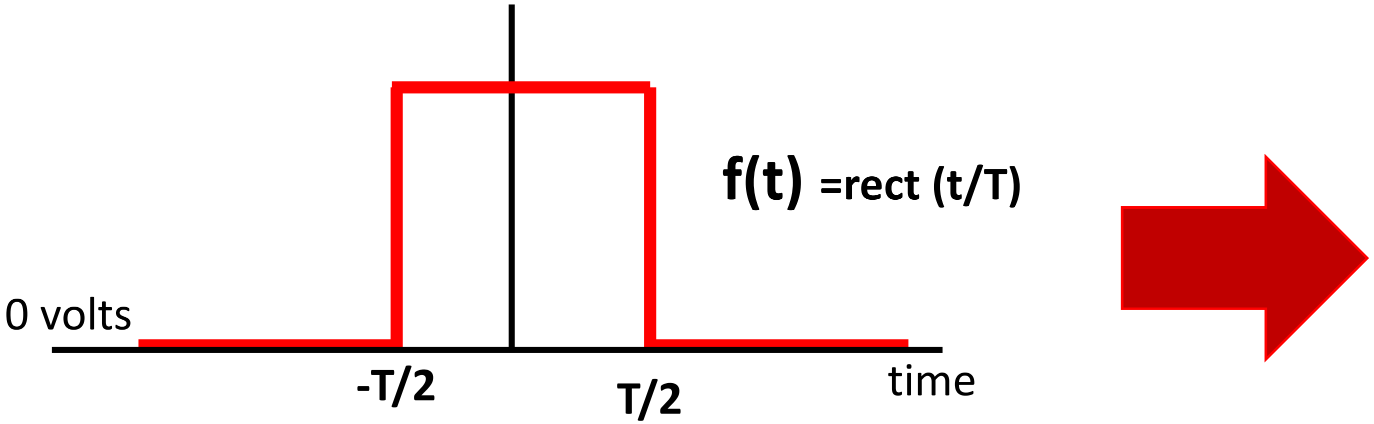
$$S_y(\omega) = \frac{|F(\omega)|^2}{T}[(1) + (0)]$$

$$S_y(\omega) = \frac{|F(\omega)|^2}{T}$$

Cool… so we just need F(w) which is our frequency pulse

Calculating the Frequency of DCS Signals

It turns out F(W) is actually one of the standard Fourier transform pairs… usually on the Inside cover of most textbooks…



$$\int_{-\infty}^{\infty} rect(t/T)dt = T\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}$$

Cool… Now we've got EVERYTHING to finish this calculation

Calculating the Frequency of DCS Signals

The DCS Spectrum is: $$S_y(\omega) = \frac{|F(\omega)|^2}{T}$$

$$S_y(\omega) = \frac{1}{T}\left|T\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}\right|^2$$

$$S_y(\omega) = \frac{T}{2}\frac{\sin(\frac{\omega T}{2})^2}{\frac{\omega T}{2}^2}$$

And finally we can use the sinc function to make it look nicer

$$S_y(\omega) = \frac{T}{2}sinc^2(\frac{\omega T}{2})$$

Spectrum of the DCS signal

Calculating the Frequency of DCS Signals

$$S_y(\omega) = \frac{T}{2} sinc^2(\frac{\omega T}{2})$$

Spectrum of the DCS signal

Remember for DCS…
T = 1/3.75 MHz = 266ns

- So the signal actually occupies all the way up to 7.5 MHz even though the Spreading-chip rate is only 3.75 MHz… (actually a tiny amount of sideband power Is up at 11.25 MHz)

- Even though there is no DC (the signal average is zero) there is actually a big DC component still… so you can’t actually AC couple the signal or you’ll filter out all that center lobe energy.